Spurious deadweight gains

Giovanni Facchini, Peter J. Hammond*, Hiroyuki Nakata

Stanford University, Department of Economics, Stanford, CA 94305-6072, USA

Abstract

Marshallian consumer surplus (MCS) is generally an inaccurate measure of welfare change because it neglects income effects. Suppose these effects overturn the usual demand response to a price change. Then, the deadweight loss from a distortionary tax or subsidy has the wrong sign, that is, there is a spurious deadweight gain. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

It is well known that Marshallian consumer surplus (MCS) is an exact measure of welfare change only when income effects can be ignored, e.g. when the income elasticity of demand is zero. Nevertheless, Robert Willig (1976) gave conditions for MCS to be a good approximation. Later, Jerry Hausman (1981) pointed out that more stringent conditions must be satisfied if a 'Marshallian' measure of deadweight loss based on MCS is to be a good approximation to the true deadweight loss.

The purpose of this note is to show that the neglected income effect can be so large that a distortionary tax or subsidy appears to provide a (spurious) deadweight gain. To simplify the argument, we consider a commodity whose aggregate demand and aggregate supply function are both linear functions of its own price (of course, the results can be generalized trivially for non-linear cases). More specifically, suppose aggregate demand is given by the function $D(p) = a - bp$ and aggregate supply is given by $S(p) = -c + dp$, with $D(p) \geq 0$ and $S(p) \geq 0$ for all $p > 0$ around the equilibria. Namely, we assume positive demand and supply with a positive price around the equilibria. Then the original equilibrium price and quantity are $p^* = (a + c)/(b + d) > 0$ and $D(p^*) = S(p^*) =$

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*Corresponding author. Tel.: +1-650-723-3987; fax: +1-650-725-5702.

E-mail address: hammond@leland.stanford.edu (P.J. Hammond).

$(ad - bc)/(b + d) \geq 0$, respectively. Now, we can distinguish the following two cases: (a) consumers pay a tax t on each unit sold ($t > 0$), (b) consumers receive a subsidy s on each unit sold ($s > 0$). Clearly, the latter case is a mirror image of the first case (by letting $t = -s$). Therefore, we can claim that the same results hold for both cases, although we shall examine only the case of tax below.

Suppose consumers pay a specific tax t on each unit sold. Then the new equilibrium consumer price is higher than the equilibrium producer/supplier price by the amount t. So, the equilibrium consumer price and producer price become $p_D^* = (a + c + dt)/(b + d)$ and $p_S^* = (a + c - bt)/(b + d)$, respectively, while the equilibrium quantity becomes $D(p_D^*) = S(p_S^*) = (ad - bc - bdt)/(b + d)$. The net increase in MCS is:

$$
\begin{aligned}
\Delta MCS &= \frac{1}{2} \cdot (p^* - p_D^*) \cdot [D(p^*) + D(p_D^*)] \\
&= \frac{1}{2} \cdot \frac{(-dt)}{b + d} \cdot \frac{2ad - 2bc - bdt}{b + d} \\
&= \frac{dt}{2(b + d)^2} (bdt + 2bc - 2ad)
\end{aligned}
\tag{1}
$$

Note that $\Delta MCS < 0$ if $d > 0$, but $\Delta MCS > 0$ if $d < 0$. On the other hand, the net gain in producer surplus is:

$$
\begin{aligned}
\Delta PS &= \frac{1}{2} \cdot (p_S^* - p^*) \cdot [S(p^*) + S(p_S^*)] \\
&= \frac{1}{2} \cdot \frac{(-bt)}{b + d} \cdot \frac{2ad - 2bc - bdt}{b + d} \\
&= \frac{bt}{2(b + d)^2} (bdt + 2bc - 2ad)
\end{aligned}
\tag{2}
$$

Hence, $\Delta PS < 0$ if $b > 0$, but $\Delta PS > 0$ if $b < 0$. Moreover, the tax revenue received by the government (TR) is:

$$
TR = t \cdot D(p_D^*) = t \cdot \frac{ad - bc - bdt}{b + d}
\tag{3}
$$

We are by now ready to calculate the 'Marshallian' deadweight loss, which is:

$$
\begin{aligned}
MDWL &= -TR - \Delta MCS - \Delta PS \\
&= -t \cdot \frac{ad - bc - bdt}{b + d} - \frac{t}{2(b + d)} (bdt + 2bc - 2ad) \\
&= \frac{bdt^2}{2(b + d)}
\end{aligned}
\tag{4}
$$

Now we present an example where a distortionary tax (or subsidy) appears to provide a (spurious) deadweight gain. Assume that $0 > b > -d$, where $b < 0$ indicates that we are considering a Giffen good. Then, by using the above formulae it is straightforward to see that $\Delta MCS < 0$, $\Delta PS > 0$ and $MDWL < 0$ hold. According to the surplus analysis here, the net decrease in consumer surplus is smaller than the sum of the net increase in producer surplus and the tax revenue; thus, it appears that there is a 'gain'. However, $-\Delta MCS$ underestimates the true loss of consumer welfare by neglecting
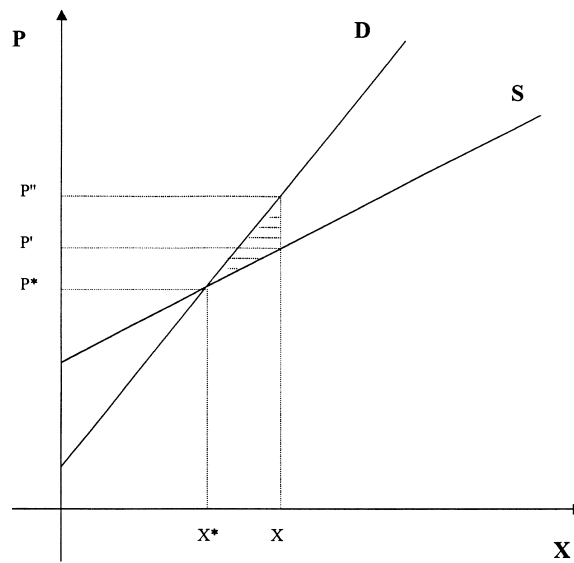
Fig. 1. Tax.

the income effect. For a geometrical representation, we provide Fig. 1 for a tax, and Fig. 2 for a subsidy.

The above example clearly shows that the Marshallian analysis fails to indicate the correct sign of the total welfare change caused by a distortionary tax or subsidy whenever the aggregate demand curve has a steeper positive slope than the supply curve (i.e. aggregate supply is more sensitive to price changes). In effect, this is because of a 'sign reversal' in the (Marshallian) measure of net deadweight loss when the aggregate demand becomes perfectly inelastic.
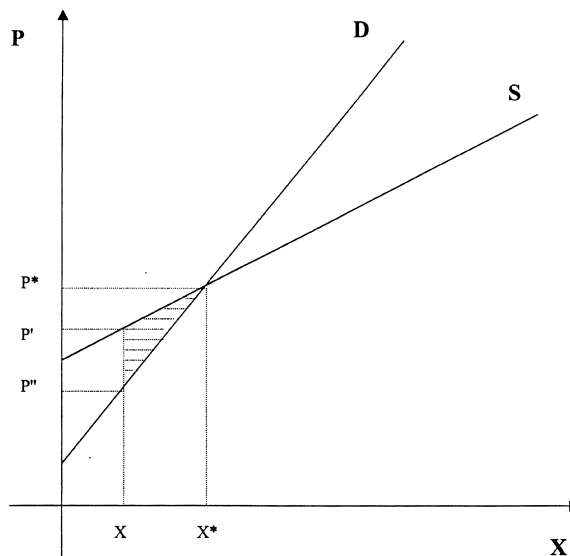

Fig. 2. Subsidy.

Table 1
Summary

Conditions	Description	Stability	Signs		
			bd	$b + d$	$MDWL$
$b > 0$ and $d > 0$	Usual	Stable	+	+	+
$0 > b > -d$	Giffen	Stable	−	+	−
$0 > -d > b$	Giffen	Unstable	−	−	+
$0 > -b > d$	BBS	Unstable	−	−	+
$0 > d > -b$	BBS	Stable	−	+	−
$b < 0$ and $d < 0$	Perverse	Unstable	+	−	−

To understand this 'sign-reversal' better, we examine all possible cases by altering the combinations (i.e. signs and relative magnitudes) of the parameters b and d. In so doing, we pay attention to the stability of the price system as well. That is, we are going to check whether excess demand forces the price to move up, and/or whether excess supply puts negative pressure on the price. Observe that the example we examined above $(0 > b > -d)$ is a stable system. It is clear from the earlier formulae that the price system is stable whenever $b + d > 0$ holds.

By applying the formulae above, we list all possible combinations of b and d in Table 1. In particular, the case of 'backward-bending supply' (BBS) arises when $0 > -b > d$ or $0 > d > -b$. Also, the last 'perverse' case arises when a Giffen good has a backward-bending supply curve.

Amongst stable systems, the sign of $MDWL$ changes when $b = 0$ (i.e. aggregate demand is inelastic) and $d > 0$, while it has a similar property when $d = 0$ (i.e. aggregate supply is inelastic) and $b > 0$. In other words, whenever the system is a regular one (i.e. aggregate demand is decreasing in price and aggregate supply is increasing in price) but either demand or supply is nearly price inelastic, the sign of $MDWL$ is prone to error. Of course, if $b = 0$ or $d = 0$, because demand or supply is perfectly inelastic, then $DWL = 0$, which is the true measure of deadweight loss allowing for income effects. The problem is that using MCS leads to $MDWL$ changing sign with b or d, whereas a true measure of deadweight loss is minimized at zero when $b = 0$ or $d = 0$. Even in the usual case, however, the magnitude of $MDWL$ may be very inaccurate although the sign is correct; thus, $MDWL$ is inappropriate if one wishes to compare welfare effects of two distinct policies in such a case.[1] Hence, we can claim that $MDWL$ is inappropriate for many practical situations.

Finally, we suggest that essentially the same results carry through when there are (physical) endowments. As is well known, introducing a distortionary tax (or subsidy) causes the budget line to pivot about the consumer's endowment. This creates an endowment effect on demand or supply in addition to an income effect. Nevertheless, these effects are already allowed for in the Marshallian demand schedules. Since our results solely rest on the signs and magnitudes of the slopes of aggregate demand and supply, the same argument holds even if there are (physical) endowments. Indeed, b or d may change sign exactly because of the endowment effect.

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[1] Hausman (1981) investigates this point by referring to the magnitude of income effects.

To conclude, this note repeats the caveat against the thoughtless use of Marshallian consumer surplus.

References

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